Section 3.6 Mathematics of Finance

Calculating the Amount in an account for Compound Interest Paid *n* times a year

If you deposit P dollars at a rate r (in decimal form) subject to compound interest paid n times a year, then the amount, A, of money in the account after t years is given by

$$A = P\left(1+\frac{r}{n}\right)^{(nt)}$$ ← n·t → # of times interest is compounded

Annually n= 1 Semi-Annually n= 2 Quarterly n= 4 Monthly n= 12 Weekly n= 52

" every 6 mo. " every 3 mo.

Examples:

1. You deposit $1000 in a savings account at a bank that has a rate of 0.95%. r = 0.0095

   P t =

   a. Find the amount, A, of money in the account after five years, subject to interest compounded annually. Round to the nearest cent. n = 1

   t = # of years
   A = total amount of money in $

   $$A = 1000\left(1+\frac{0.0095}{1}\right)^{(1\cdot5)} = \boxed{\$1048.41}$$

   b. Find the interest after five years.

   1048.41 − 1000 $\boxed{\$48.41}$

2. You deposit $4200 in a savings account that has a rate of 1.125%. The interest is compounded quarterly. P r = 0.01125 n = 4

   a. How much money will you have after ten years? Round to the nearest cent. t =

   $$A = 4200\left(1+\frac{0.01125}{4}\right)^{(4\cdot10)} = \boxed{\$4699.36}$$

   b. Find the interest after ten years.

   $4699.36 - 4200 = \boxed{\$499.36}$

3. How much money should be deposited today in an account that earns 7% compounded monthly, so that it will accumulate to $10,000 in eight years? P = ? A

   $$\underbrace{10000}_{ANS} = P\underbrace{\left(1+\frac{0.07}{12}\right)^{(12\cdot8)}}_{ANS}$$

   $P = \boxed{\$5721.39}$

4. If John invests $2300 in a savings account with a 9% interest rate compounded quarterly, how long will it take until John's account has a balance of $4150? 4t

   $$\frac{4150}{2300} = \frac{2300}{2300}\left(1+\frac{0.09}{4}\right)^{4t}$$

   $$LN\left(\frac{4150}{2300}\right) = LN\left(1+\frac{0.09}{4}\right)^{4t}$$

   $$\frac{LN(4150/2300)}{LN\left(1+\frac{0.09}{4}\right)} = \frac{4t\ Ln\left(1+\frac{0.09}{4}\right)}{Ln\left(1+\frac{0.09}{4}\right)}$$

   $$\frac{26.525\ldots}{4} = \frac{4t}{4}$$

   $\boxed{t = 6.6313 \text{ years}}$

5. What interest rate compounded daily (365 days/year) is required for a $22,000 investment to grow to $36,500 in 5 years?

   $$\frac{36500}{22000} = \frac{22000}{22000}\left(1+\frac{r}{365}\right)^{(365\cdot5)}$$

   $$\sqrt[1825]{\frac{365}{220}} = \sqrt[1825]{\left(1+\frac{r}{365}\right)^{(1825)}}$$

   $$1.0002777447 = 1+\frac{r}{365}$$

   $0.101268\ldots = r$

   $\boxed{10.13\%}$

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Calculating the Amount in an account when Compounding Interest Continuously
If you deposit P dollars at a rate r (in decimal form) and the interest is compounded continuously, then the amount, A, of money in the account after t years is given by

$$A = Pe^{(rt)}$$ where $e \approx 2.71828$ (use the e^x button on your calculator)

Examples:

6. A single payment of $10,000 is invested at an annual rate of 8%, compounded continuously. Find the balance in the account after 5 years.

P   r = 0.08 Pe^{rt}

$A = 10{,}000\, e^{\wedge(0.08 \times 5)}$

$\boxed{\$14{,}918.25}$

7. You have deposited $1000 in an account that pays 6.25% interest compounded continuously. How long will it take for your money to double?

$\frac{2000}{1000} = \frac{1000}{1000} e^{0.0625t}$

$\ln 2 = \ln e^{0.0625t}$

$\ln 2 = 0.0625t \underbrace{\ln e}_{1}$

$\frac{\ln(2)}{0.0625} = \frac{0.0625t}{0.0625}$

11.0904 = t years

8. When your initial investment is earning interest in an account with continuous compounding, find the missing information given the various conditions.

Pe^{rt}

a) Initial Investment: $12,500 (P) APR 9% ($r = 0.09$) Time to double your money? Amount in 15 years?

$25000 = 12500\, e^{0.09t}$

$\ln 2 = \ln e^{0.09t}$

$\frac{\ln 2}{0.09} = \frac{0.09t}{0.09} \ln e$

$t = 7.7016413$

$A = 12500\, e^{(0.09 \times 15)}$

$\boxed{\$48217.82}$

b) Initial Investment: $9,500 (P) APR ? (r = ?) Time to double your money: 4 years Amount in 15 years?

$19000 = 9500\, e^{4r}$

$\ln 2 = \ln e^{4r}$

$\boxed{r = \frac{\ln(2)}{4}} = \frac{4r \ln e}{4}$

$0.1733 = r$ 17.33%

$A = 9500\, e^{\wedge\left(\frac{\ln(2)}{4} \times 15\right)}$

$\boxed{\$127{,}816.26}$

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